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Mathematics in the streets and in schools

Terezinha Nunes Carraher; David William Carraher and Analucia Dias Schliemann

An analysis of everyday use of mathematics by working youngsters in commercial transactions in Recife, Brazil, revealed computational strategies different from those taught in schools. Performance on mathematical problems embedded in real-life contexts was superior to that on school-type word problems and context-free computational problems involving the same numbers and operations. Implications for education are examined.

There are reasons for thinking that there may be a difference between solving mathematical problems using algorithms learned in school and solving them in familiar contexts out of school. Reed & Lave (1981) have shown that people who have not been to school often solve such problems in different ways from people who have. This certainly suggests that there are informal ways of doing mathematical calculations which have little to do with the procedures taught in school.

Reed & Lave's study with Liberian adults showed differences between people who had and who had not been to school. However, it is quite possible that the same differences between informal and school-based routines could exist within people. In other words it might be the case that the same person could solve problems sometimes in formal and at other times in informal ways. This seems particularly likely with children who often have to do mathematical calculations in informal circumstances outside school at the same time as their knowledge of the algorithms which they have to learn at school is imperfect and their use of them ineffective.

We already know that children often obtain absurd results such as finding a remainder which is larger than the minuend when they try to apply routines for computations which they learn at school (Carraher & Schliemann, in press). There is also some evidence that informal procedures learned outside school are often extremely effective. Gay & Cole (1976) for example showed that unschooled Kpelle traders estimated quantities of rice far better than educated Americans managed to. So it seems quite possible that children might have difficulty with routines learned at school and yet at the same time be able to solve the mathematical problems for which these routines were devised in other more effective ways. One way to test this idea is to look at children who have to make frequent and quite complex calculations outside school. The children who sell things in street markets in Brazil form one such group (Carraher *et al.*, 1982).

The cultural context

The study was conducted in Recife, a city of approximately 1.5 million people on the north-eastern coast of Brazil. Like several other large Brazilian cities, Recife receives a very large number of migrant workers from the rural areas who must adapt to a new way of living in a metropolitan region. In an anthropological study of migrant workers in Sao Paulo, Brazil, Berlinck (1977) identified four pressing needs in this adaptation process: finding a home, acquiring work papers, getting a job, and providing for immediate survival (whereas in rural areas the family often obtains food through its own work). During the initial adaptation phase, survival depends mostly upon resources brought by the migrants or received through begging. A large portion of migrants later become unspecialized manual workers, either maintaining a regular job or working in what is known as the informal sector of the economy (Cavalcanti, 1978). The informal sector can be characterized as an unofficial part of the economy which consists of relatively unskilled jobs not regulated by government organs thereby producing income not susceptible to taxation while at the same time not affording job security or workers' rights such as health insurance. The income generated thereby is thus intermittent and variable. The dimensions of a business enterprise in the informal sector are determined by the family's work capability. Low educational and professional qualification levels are characteristic of the rather sizable population which depends upon the informal sector. In Recife, approximately 30 per cent of the workforce is engaged in the informal sector as its main activity and 18 per cent as a secondary activity (Cavalcanti, 1978). The importance of such sources of income for families in Brazil's lower socio-economic strata can be easily understood by noting that the income of an unspecialized labourer's family is increased by 56 percent through his wife's and children's activities in the informal sector in Sao Paulo; (Berlinck, 1977). In Fortaleza it

represents fully 60 per cent of the lower class* family's income (Cavalcanti & Duarte, 1980a).

Several types of occupations—domestic work, street-vending, shoe-repairing and other types of small repairs which are carried out without a fixed commercial address—are grouped as part of the informal sector of the economy. The occupation considered in the present study—that of street-vendors—represents the principal occupation of 10 per cent of the economically active population of Salvador (Cavalcanti & Duarte, 1980b) and Fortaleza (Cavalcanti & Duarte, 1980a). Although no specific data regarding street-vendors were obtained for Recife, data from Salvador and Fortaleza serve as close approximations since these cities are, like Recife, State capitals from the same geographical region.

It is fairly common in Brazil for sons and daughters of street-vendors to help out their parents in their businesses. From about the age of 8 or 9 the children will often enact some of the transactions for the parents when they are busy with another customer or away on some errand. Pre-adolescents and teenagers may even develop their own 'business', selling snack foods such as roasted peanuts, pop-corn, coconut milk or corn on the cob. In Fortaleza and Salvador, where data are available, 2.2 and 1.4 per cent, respectively, of the population actively engaged in the informal sector as street-vendors were aged 14 or less while 8.2 and 7.5 percent, respectively, were aged 15-19 years (Cavalcanti & Duarte, 1980a,b).

In their work these children and adolescents have to solve a large number of mathematical problems, usually without recourse to paper and pencil. Problems may involve multiplication (one coconut cost x ; four coconuts, $4x$), addition (4 coconuts and 12 lemons cost $x + y$), and subtraction (Cr\$500—i.e. 500 *cruzeiros*—minus the purchase price will give the change due). Division is much less frequently used but appears in some contexts in which the price is set with respect to a measuring unit (such as 1 kg) and the customer wants a fraction of that unit: *for example*, when the particular item chosen weighs 1.2 kg. The use of tables listing prices by number of items (one egg-12 *cruzeiros*; two eggs-24, etc.) is observed occasionally in natural settings but was not observed among the children who took part in the study. Pencil and paper were also not used by these children, although they may occasionally be used by adult vendors when adding long lists of items.

Method

Subjects

The children in this study were four boys and one girl aged 9-15 years with a mean age of 11.2 and ranging in level of schooling from first to eighth grade. One of them had only one year of schooling; two had three years of schooling; one, four years; and one, eight years. All were from very poor backgrounds. Four of the subjects were attending school at the time and one had been out of school for two years. Four of these subjects had received formal instruction on mathematical operations and word problems. The subject who attended first grade and dropped out of school was unlikely to have learned multiplication and division in school since these operations are usually initiated in second or third grade in public schools in Recife,

Procedure

The children were found by the interviewers on street corners or at markets where they worked alone or with their families. Interviewers chose subjects who seemed to be in the desired age range—school children or young adolescents—obtaining information about their age and level of schooling along with information on the prices of their merchandise. Test items in this situation were presented in the course of a normal sales transaction in which the researcher posed as a customer. Purchases were sometimes carried out. In other cases the 'customer' asked the vendor to perform calculations on possible purchases. At the end of the informal test, the children were asked to take part in a formal test which was given on a separate occasion, no more than a week later, by the same interviewer. Subjects answered a total of 99 questions on the formal test and 63 questions on the informal test. Since the items of the formal test were based upon questions of the informal test, order of testing was fixed for all subjects.

* In the present report the term 'class' is employed loosely, without a clear distinction from the expression 'socio-economic stratum'.

(1) *The informal test*, The informal test was carried out in Portuguese in the subject's natural working situation, that is, at street corners or an open market. Testers posed to the subject successive questions about potential or actual purchases and obtained verbal responses. Responses were either tape-recorded or written down, along with comments, by an observer. After obtaining an answer for the item, testers questioned the subject about his or her method for solving the problem.

The method can be described as a hybrid between the Piagetian clinical method and participant observation. The interviewer was not merely an interviewer; he was also a customer—a questioning customer who wanted the vendor to tell him how he or she performed their computations.

An example is presented below taken from the informal test with M., a coconut vendor aged 12, third grader, where the interviewer is referred to as 'customer':

Customer: How much is one coconut?

M: 35.

Customer: I'd like ten. How much is that?

M. (Pause) Three will be 105; with three more, that will be 210. (Pause) I need four more. That is...
(pause) 315... I think it is 350.*

This problem can be mathematically represented in several ways: 35×10 is a good representation of the *question* posed by the interviewer. The subject's answer is better represented by $105 + 105 + 105 + 35$, which implies that 35×10 was solved by the subject as $(3 \times 35) + (3 \times 35) + (3 \times 35) + 35$. The subject can be said to have solved the following subitems in the above situation:

(a) 35×10 ;

(b) 35×3 (which may have already been known);

(c) $105 + 105$;

(d) $210 + 105$;

(e) $315 + 35$;

(f) $3 + 3 + 3 + 1$.

When one represents in a formal mathematical fashion the problems which were solved by the subject, one is in fact attempting to represent the subject's mathematical competence. M. proved to be competent in finding out how much 35×10 is, even though he used a routine not taught in third grade, since in Brazil third-graders learn to multiply any number by ten simply by placing a zero to the right of that number. Thus, we considered that the subject solved the test item (35×10) and a whole series of sub-items (*b* to *f*) successfully in this process. However, In the process of scoring, only *one* test item (35×10) was considered as having been presented and, therefore, correctly solved.

(2) *The formal test*. After subjects were interviewed in the natural situation, they were asked to participate in the formal part of the study and a second interview was scheduled at the same place or at the subject's house.

The items for the formal test were prepared for each subject on the basis of problems solved by him or her during the informal test. Each problem solved in the informal test was mathematically represented according to the subject's problem-solving routine.

From all the mathematical problems *successful/y solved* by each subject (regardless of whether they constituted a test item or not), a sample was chosen for inclusion in the subject's formal test. This sample was presented in the formal test either as a mathematical operation dictated to the subject (e.g. $105 + 105$) or as a word problem e.g. Mary bought x bananas; each banana cost y ; how much did she pay altogether?). In either case, *each subject solved problems employing the same numbers involved in his or her own informal test*. Thus quantities used varied from one subject to the other.

Two variations were introduced in the formal test, according to methodological suggestions contained in [Reed & Lave \(1981\)](#). First, some of the items presented in the formal test were the inverse of problems solved in the informal test (e.g. $500 - 385$ may be presented as $385 + 115$ in the formal test). Second, some of the

* (...) is used here to mark ascending intonation suggestive of the interruption, and not completion, of a statement.

items in the informal test used a decimal value which differed from the one used in the formal test (e.g. 40 *cruzeiros* may have appeared as 40 *centavos* or 35 may have been presented as 3500 in the formal test--the principal Brazilian unit of currency is the *cruzeiro*; each *cruzeiro* is worth one hundred *centavos*).

In order to make the formal test situation more similar to the school selling, subjects were given paper and pencil at the testing and were encouraged to use these. When problems were nonetheless solved without recourse; to writing, subjects were asked to write down their answers. Only one subject refused to do so, claiming that he did not know how to write. It will be recalled, however, that the school-type situation was not represented solely by the introduction of pencil and paper but also by the very use of formal mathematical problems without context and by word problems referring to imaginary situations.

In the formal test the children were given a total of 38 mathematical operations and 61 word problems. Word problems were rather concrete and each involved only one mathematical operation.

Results and discussion

The analysis of the results from the informal test required an initial definition of what would be considered a test item in that situation. While, in the formal test, items were defined prior to testing, in the informal test problems were generated in the natural setting and items were identified *a posteriori*. In order to avoid a biased increase in the number or; items solved in the informal test, the definition of an item was based upon *questions* posed by the customer/tester. This probably constitutes a conservative estimate of the number of problems solved, since subjects often solved a number of intermediary steps in the course of searching for the solution to the question they had been asked. Thus the same defining criterion was applied in both testing situations in the identification of items even though items were defined prior to testing in one case and after testing in the other. In both testing situations, the subject's oral response was the one taken into account even though in the formal test written responses were also available.

Context-embedded problems were much more easily solved than ones without a context. Table I shows that 98.2 per cent of the 63 problems presented in the informal test were correctly solved. In the formal test word problems (which provide some descriptive context for the subject), the rate of correct responses was 73.7 per cent, which should be contrasted with a 36.8 per cent rate of correct responses for mathematical operations with no context.

Table 1. Results according to testing conditions

Subject	Informal test		Formal Test			
	Score *	Number of items	Mathematical Operations	Word Problems	Score	Number of items
M	10	18	Score	Number of items	10	11
P	8.9	19	3.7	8	6.9	16
Pi	10	12	5.0	6	10	11
MD	10	7	1.0	10	3.3	12
S	10	7	8.3	6	7.3	11
Totals		63		38		61

*Each subject's score is the percentage of correct items divided by 10.

The frequency of correct answers for each subject was converted to scores from I to 10 reflecting the percentage of correct responses. A Friedman two-way analysis of variance or score ranks compared the scores of each subject in the three types of testing conditions. The scores differ significantly across conditions ($\chi^2 r=6.4, P=0.039$). Mann-Whitney *Us* were also calculated comparing the three types of testing situations. Subjects performed significantly better on the informal test than on the formal test involving context-free operations ($U = 0, P < 0.05$). The difference between the informal test and the word problems was not significant ($U = 6, P > 0.05$).

It could be argued that errors observed in the formal test were related to the transformations that had been performed upon the informal test problems in order to construct the formal test. An evaluation of this hypothesis was obtained by separating items which had been changed either by inverting the operation or changing the decimal point from items which remained identical to their informal test equivalents. The percentage of correct responses in these two groups of items did not differ significantly; the rate of correct responses in transformed items was slightly higher than that obtained for items identical to informal test items. Thus the transformations performed upon informal test items in designing formal test items cannot explain the discrepancy of performance in these situations. .

A second possible interpretation of these results is that the children interviewed in this study were 'concrete' in their thinking and, thus, concrete situations would help them in the discovery of a solution. In the natural situation, they solved problems about the sale of lemons, coconuts, etc., when the actual items in question were physically present. However, the presence of concrete instances can be understood as a facilitating factor if the instance somehow allows the problem-solver to abstract from the concrete example to a more general situation. There is nothing in the nature of coconuts that makes it relatively easier to discover that three coconuts (at Cr\$ 35.00 each) cost Cr\$ 105.00. The presence of the groceries does not simplify the arithmetic of the problem. Moreover, computation in the natural situation of the informal test was in all cases carried out mentally, without recourse to external memory aids for partial results or intermediary steps. One can hardly argue that mental computation would be an ability characteristic of concrete thinkers.

The results seem to be in conflict with the implicit pedagogical assumption of mathematical educators according to which children ought first to learn mathematical operations and only later to apply them to verbal and real-life problems. Real-life and word problems may provide the 'daily human sense' (Donaldson, 1978) which will guide children to find a correct solution intuitively without requiring an extra step—namely, the translation of word problems into algebraic expressions. This interpretation is consistent with data obtained by others in the area of logic, such as [Wason & Shapiro \(1971\)](#), [Johnson-Laird et al. \(1972\)](#) and [Lunzer et al. \(1972\)](#).

How is it possible that children capable of solving a computational problem in the natural situation will fail to solve the same problem when it is taken out of its context? In the present case, a qualitative analysis of the protocols suggested that the problem-solving routines used may have been different in the two situations. In the natural situations children tended to reason by using what can be termed a 'convenient group' while in the formal test school-taught routines were more frequently, although not exclusively, observed. Five examples are given below, which demonstrate the children's ability to deal with quantities and their lack of expertise in manipulating symbols. The examples were chosen for representing clear explanations of the procedures used in both settings. In each of the five examples below the performance described in the informal test contrasts strongly with the same child's performance in the formal test when solving the same item.

(1) First example (M, 12 years)

Informal test

Customer: I'm going to take four coconuts. How much is that?

Child: Three will be 105, plus 30, that's 135 . . . one coconut is 35... that is . . . 140!

Formal test

Child resolves the item 35×4 explaining out loud:

4 times 5 is 20, carry the 2; 2 plus 3 is 5, times 4 is 20. Answer written: 200.

(2) Second example (MD, 9 years)

Informal test

Customer: OK, I'll take three coconuts (at the price of Cr\$ 40.00 each). How much is that?

Child: (Without gestures, calculates out loud) 40, 80, 120.

Formal test

Child solves the item 40×3 and obtains 70. She then explains the procedure 'Lower the zero; 4 and 3 is 7'.

(3) Third example (MD, 9 years)

Informal test

Customer: I'll take 12 lemons (one lemon is Cr\$ 5.00)

Child: 10, 20, 30, 40, 50, 60 (while separating out two lemons at a time).

Formal test;

Child has just solved the item 40×3 . In solving 12×5 she proceeds by lowering first the 2, then the 5 and the 1, obtaining 152. She explains this procedure to the (surprised) examiner when she is finished.

(4) Fourth example (S, II years)

Informal test

Customer: What would I have to pay for six kilos? (of watermelon at Cr\$ 50.00 per kg).

Child: [Without any appreciable pause] 300.

Customer: Let me see. How did you get that so fast?

Child: Counting one by one. Two kilos, 100. 200. 300.

Formal test

Test item: A fisherman caught 50 fish. The second one caught five times the amount of fish the first fisherman had caught. How many fish did the lucky fisherman catch?

Child: (Writes down 50×6 and 360 as the result; then answers) 36.

Examiner repeats the problems and child does the computation again, writing down 860 as result. His oral response is 86.

Examiner: How did you calculate that?

Child: I did it like this. Six times six is 36. Then I put it there.

Examiner: Where did you put it? (Child had not written down the number to be carried.)

Child: (Points to the digit 5 in 50). That makes 86 [apparently adding 3 and 5 and placing this sum in the result].

Examiner: How many did the first fisherman catch? Child: 50.

A final example follows, with suggested interpretations enclosed in parentheses.

(5) Fifth example

Informal test

Customer: I'll take two coconuts (at Cr\$ 40.00 each. Pays with a Cr\$ 500.00 bill). What do I get back?

Child: (Before reaching for customer's change) 80, 90, 100. 420.

Formal test

Test item: $420 + 80$.

The child writes $420 + 80$ and claims that 130 is the result. [The procedure used was not explained but it seems that the child applied a step of a multiplication routine to an addition problem by successively adding 8 to 2 and then to 4, carrying the 1; that is, $8 + 2 = 10$, carry the one, $1 + 4 + 8 = 13$. The zeros in 420 and 80 were not written. Reaction times were obtained from tape recordings and the whole process took 53 seconds.]

Examiner: How did you do this one, 420 plus 80?

Child: Plus?

Examiner: Plus 80.
Child: 100, 200.
Examiner: (After a 5 second pause, interrupts the child's response treating it as final)
Hum, OK.
Child: Wait a minute. That was wrong. 500. [The child had apparently added 80 and 20, obtaining one hundred, and then started adding the hundreds. The experimenter interpreted 200 as the final answer after a brief pause but the child completed the computation and gave the correct answer when solving the addition problem by a manipulation-with-quantities approach.]

In the informal test, children rely upon mental calculations which are closely linked to the quantities that are being dealt with. The preferred strategy for multiplication problems seems to consist in chaining successive additions. In the first example, as the addition became more difficult, the subject decomposed a quantity into tens and units—to add 35 to 105, M. first added 30 and later included 5 in the result.

In the formal test, where paper and pencil were used in all the above examples, the children try to follow, without success, school-prescribed routines. Mistakes often occur as a result of confusing addition routines with multiplication routines, as is clearly the case in examples (1) and (5). Moreover, in all the cases, there is no evidence, once the numbers are written down, that the children try to relate the obtained results to the problem at hand in order to assess the adequacy of their answers.

Summarizing briefly, the combination of the clinical method of questioning with participant observation used in this project seemed particularly helpful when exploring mathematical thinking and thinking in daily life. The results support the thesis proposed by Luria (1976) and by Donaldson (1978) that thinking sustained by daily human sense can be—in the same subject—at a higher level than thinking out of context. They also raise doubts about the pedagogical practice of teaching mathematical operations in a disembedded form before applying them to word problems.

Our results are also in agreement with data reported by Lave *et al.* (1984), who showed that problem solving in the supermarket was significantly superior to problem solving with paper and pencil. It appears that daily problem solving may be accomplished by routines different from those taught in schools. In the present study, daily problem solving tended to be accomplished by strategies involving the mental manipulation of quantities while in the school-type situation the manipulation of symbols carried the burden of computation, thereby making the operations 'in a very real sense divorced from reality' (see [Reed & Lave, 1981](#), p. 442). In many cases attempts to follow school-prescribed routines seemed in fact to interfere with problem solving (see also Carraher & Schliemann, in press).

Are we to conclude that schools ought to allow children simply to develop their own computational routines without trying to impose the conventional systems developed in the culture? We do not believe that our results lead to this conclusion. Mental computation has limitations which can be overcome through written computation. One is the inherent limitation placed on multiplying through successive chunking, that is, on multiplying through repeated chunked additions—a procedure which becomes grossly inefficient when large numbers are involved.

The sort of mathematics taught in schools has the potential to serve as an 'amplifier of thought processes', in the sense in which Bruner (1972) has referred to both mathematics and logic. As such, we do not dispute whether 'school maths' routines can offer richer and more powerful alternatives to maths routines which emerge in non-school settings. The major question appears to centre on the proper pedagogical point of departure, i.e. where to start. We suggest that educators should question the practice of treating mathematical systems as formal subjects from the outset and should instead seek ways of introducing these systems in contexts which allow them to be sustained by human daily sense.

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Requests for reprints should be addressed to Dr T. N. Carraher. Universidade Federal de Pernambuco, Rua Mendes Martins, 112, Várzea, 50 000 Recife PE, Brazil.

D. W. Carraher and A. D. Schliemann are also at the Universidade Federal de Pernambuco, Recife.